# Pracovní list 02 – LIMITA FUNKCE II

Vypočtěte, pokud existují, následující limity:

## Příklad 17.:

$$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x}\right)\left(\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}\right)}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}=$$

$$=\lim_{x\to\infty}\frac{x+\sqrt{x+\sqrt{x}}-x}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x}}=\lim_{x\to\infty}\frac{\sqrt{x}\sqrt{\frac{x+\sqrt{x}}{x}}}{\sqrt{x+\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}}}+\sqrt{x}}=$$

$$=\lim_{x\to\infty}\frac{\sqrt{x}\sqrt{\frac{x}{x}+\frac{\sqrt{x}}{x}}}{\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}}+\sqrt{x}}=\lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{1+\frac{1}{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}}+1}=$$

$$=\frac{\sqrt{1+0}}{\sqrt{1+0\sqrt{1+0}}+1}=\underline{\underline{\frac{1}{2}}}$$

## Příklad 18.:

Vypočtěte limitu:

$$\lim_{x\to\infty}\left(\sqrt{x+\sqrt{x+\sqrt{x}}}-\sqrt{x-\sqrt{x+\sqrt{x}}}\right)=\lim_{x\to\infty}\frac{x+\sqrt{x+\sqrt{x}}-x+\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x+\sqrt{x}}}+\sqrt{x-\sqrt{x+\sqrt{x}}}}=$$

$$=\lim_{x\to\infty}\frac{2\sqrt{x+\sqrt{x}}}{\sqrt{x+\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}}}+\sqrt{x-\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}}}}=$$

$$=\lim_{x\to\infty}\frac{2\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}}}{\sqrt{x}\sqrt{1+\frac{1}{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}}+\sqrt{x}\sqrt{1-\frac{1}{\sqrt{x}}\sqrt{1+\frac{1}{\sqrt{x}}}}}=$$

$$=\frac{2\sqrt{1+0}}{\sqrt{1+0\sqrt{1+0}}+\sqrt{1-0\sqrt{1+0}}}=\frac{2}{2}=\underline{\underline{1}}$$

# Pracovní list 02 – LIMITA FUNKCE II

Vypočtěte, pokud existují, následující limity:

## Příklad 12.:

$$\lim_{x\to\infty} x\left(\sqrt{x^2+2x}-2\sqrt{x^2+x}+x\right)=$$

$$=\lim_{x\to\infty} x\left(\sqrt{x^2+2x}-\sqrt{x^2+x}-\sqrt{x^2+x}+x\right)=$$

$$=\lim_{x\to\infty} x\left(\frac{\left(\sqrt{x^2+2x}-\sqrt{x^2+x}\right)\left(\sqrt{x^2+2x}+\sqrt{x^2+x}\right)}{\sqrt{x^2+2x}+\sqrt{x^2+x}}+\frac{\left(x-\sqrt{x^2+x}\right)\left(x+\sqrt{x^2+x}\right)}{x+\sqrt{x^2+x}}\right)=$$

$$=\lim_{x\to\infty} x\left(\frac{x^2+2x-x^2-x}{\sqrt{x^2+2x}+\sqrt{x^2+x}}+\frac{x^2-x^2-x}{x+\sqrt{x^2+x}}\right)=\lim_{x\to\infty} x\left(\frac{x}{\sqrt{x^2+2x}+\sqrt{x^2+x}}-\frac{x}{x+\sqrt{x^2+x}}\right)=$$

$$=\lim_{x\to\infty} x\frac{x\left(x+\sqrt{x^2+x}\right)-x\left(\sqrt{x^2+2x}+\sqrt{x^2+x}\right)}{\left(\sqrt{x^2+2x}+\sqrt{x^2+x}\right)\left(x+\sqrt{x^2+x}\right)}=\lim_{x\to\infty} x\frac{x^2-x\sqrt{x^2+2x}}{\left(\sqrt{x^2+2x}+\sqrt{x^2+x}\right)\left(x+\sqrt{x^2+x}\right)}=$$

$$=\lim_{x\to\infty} x\frac{x^2-x\sqrt{x^2+2x}}{x\left(\sqrt{1+\frac{2}{x}}+\sqrt{1+\frac{1}{x}}\right)\left(x+x\sqrt{1+\frac{1}{x}}\right)}=\lim_{x\to\infty}\frac{x-\sqrt{x^2+2x}}{\left(\sqrt{1+\frac{2}{x}}+\sqrt{1+\frac{1}{x}}\right)\left(1+\sqrt{1+\frac{1}{x}}\right)}=$$

$$=\lim_{x\to\infty}\frac{\left(x-\sqrt{x^2+2x}\right)\left(x+\sqrt{x^2+2x}\right)}{\left(\sqrt{1+\frac{2}{x}}+\sqrt{1+\frac{1}{x}}\right)\left(1+\sqrt{1+\frac{1}{x}}\right)\left(x+\sqrt{x^2+2x}\right)}=$$

$$=\lim_{x\to\infty}\frac{-2x}{\left(\sqrt{1+\frac{2}{x}}+\sqrt{1+\frac{1}{x}}\right)\left(1+\sqrt{1+\frac{1}{x}}\right)x\left(1+\sqrt{1+\frac{2}{x}}\right)}=$$

$$=\frac{-2}{\left(\sqrt{1+0}+\sqrt{1+0}\right)\left(1+\sqrt{1+0}\right)\left(1+\sqrt{1+0}\right)}=\frac{-2}{2\cdot 2\cdot 2}=\underline{\underline{-\frac{1}{4}}}$$