1. Najděte primitivní funkci k funkci $y = \dfrac{x^3+1}{x^3-5x^2+6x}$ a určete její D_f.

$$\int \frac{x^3+1}{x^3-5x^2+6x}dx = \int\left(1+\frac{5x^2-6x+1}{x(x-2)(x-3)}\right)dx \qquad D_f \in R-\{0,2,3\}$$

$$\frac{5x^2-6x+1}{x^3-5x^2+6x} = \frac{A}{x}+\frac{B}{x-2}+\frac{C}{x-3}$$

$$5x^2-6x+1 = A\left(x^2-5x+6\right)+B\left(x^2-3x\right)+C\left(x^2-2x\right)$$

$$5x^2-6x+1 = x^2(A+B+C)-x(5A+3B+2C)+6A$$

$$5 = A+B+C$$

$$6 = 5A+3B+2C$$

$$1 = 6A$$

$$A=\frac{1}{6} \qquad B=-\frac{27}{6} \qquad C=\frac{28}{3}$$

$$\int \frac{x^3+1}{x^3-5x^2+6x}dx = \int dx+\frac{1}{6}\int\frac{1}{x}dx-\frac{27}{6}\int\frac{1}{x-2}dx+\frac{28}{3}\int\frac{1}{x-3}dx =$$

$$= x+\frac{1}{6}\ln|x|-\frac{9}{2}\ln|x-2|+\frac{28}{3}\ln|x-3|+C$$

2. Najděte primitivní funkci k funkci $y=\dfrac{x^2+4x+4}{x^3-2x^2+x}$ a určete její D_f.

$$\int \frac{x^2+4x+4}{x^3-2x^2+x}\,dx = \qquad\qquad D_f \in R-\{0,1\}$$

$$\left[\begin{array}{l}
\dfrac{x^2+4x+4}{x^3-2x^2+x}=\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}\\
x^2+4x+4=A\left(x^2-2x+1\right)+B\left(x^2-x\right)+Cx\\
x^2+4x+4=x^2(A+B)+x(-2A-B+C)+A\\
\hline
1=A+B\\
4=-2A-B+C\\
4=A\\
\hline
A=4\\
B=-3\\
C=9
\end{array}\right]$$

$$\int \frac{x^2+4x+4}{x^3-2x^2+x}\,dx = 4\int\frac{1}{x}\,dx-3\int\frac{1}{x-1}\,dx+9\int\frac{1}{(x-1)^2}\,dx^{(\downarrow)} =$$

$$=4\ln|x|-3\ln|x-1|-\frac{9}{x-1}+C$$

$$^{(\downarrow)}\int\frac{1}{(x-1)^2}\,dx=\left[\begin{array}{r} x-1=t\\ dx=dt\end{array}\right]=\int\frac{1}{t^2}\,dt=-\frac{1}{t}=-\frac{1}{x-1}+c$$

3. Najděte primitivní funkci k funkci $y=\frac{4x^2+5}{x^3-2x^2+5x}$ a určete její D_f.

$$\int \frac{4x^2+5}{x^3-2x^2+5x}dx = \qquad\qquad D_f \in R-\{0\}$$

$$\left[\begin{array}{c} \frac{4x^2+5}{x^3-2x^2+5x}=\frac{A}{x}+\frac{Bx+C}{x^2-2x+5} \\ 4x^2+5=A(x^2-2x+5)+Bx^2+Cx \\ 4x^2+5=x^2(A+B)+x(C-2A)+5A \\ \hline 4=A+B \\ 0=C-2A \\ 5=5A \\ \hline A=\underline{1} \qquad B=\underline{3} \qquad C=\underline{2} \end{array}\right]$$

$$\int \frac{4x^2+5}{x^3-2x^2+5x}dx=\int\frac{1}{x}dx+\int\frac{3x+2}{x^2-2x+5}dx=\int\frac{1}{x}dx+3\int\frac{x}{x^2-2x+5}dx+2\int\frac{1}{x^2-2x+5}dx=$$

$$=\int\frac{1}{x}dx+\frac{3}{2}\int\frac{2x-2+2}{x^2-2x+5}dx+2\int\frac{1}{x^2-2x+5}dx=$$

$$=\int\frac{1}{x}dx+\frac{3}{2}\int\frac{2x-2}{x^2-2x+5}dx+3\int\frac{1}{x^2-2x+5}dx+2\int\frac{1}{x^2-2x+5}dx^{(\downarrow)}=$$

$$=\underline{\underline{\ln|x|+\frac{3}{2}\ln|x^2-2x+5|+\frac{5}{2}arctg\frac{x+1}{2}+C}}$$

$$^{(\downarrow)}\int\frac{1}{x^2-2x+5}dx=\frac{1}{\sqrt{5-\left(\frac{2}{2}\right)^2}}arctg\frac{x+\frac{2}{2}}{\sqrt{5-\left(\frac{2}{2}\right)^2}}+c=\frac{1}{2}arctg\frac{x+1}{2}+c$$

4. Najděte primitivní funkci k funkci $y=\dfrac{x^2+2x}{\left(x^2+2x+2\right)^2}$ a určete její D_f.

$$y=\frac{x^2+2x}{\left(x^2+2x+2\right)^2} \qquad D_f \in R$$

$$\int\frac{x^2+2x}{\left(x^2+2x+2\right)^2}dx=\int\frac{x^2+2x+2-2}{\left(x^2+2x+2\right)^2}dx=\int\frac{x^2+2x+2}{\left(x^2+2x+2\right)^2}dx-\int\frac{2}{\left(x^2+2x+2\right)^2}dx=$$

$$=\int\frac{1}{x^2+2x+2}dx^{*}-2\int\frac{1}{\left(x^2+2x+2\right)^2}dx^{**}=\underline{\underline{-\frac{x+1}{x^2+2x+2}+c}}$$

$$*\int\frac{1}{x^2+2x+2}dx=\frac{1}{\sqrt{2-\left(\frac{2}{2}\right)^2}}arctg\frac{x+\frac{2}{2}}{\sqrt{2-\left(\frac{2}{2}\right)^2}}+c_1=\underline{\underline{arctg(x+1)+c_1}}$$

$$**\,2\int\frac{1}{\left(x^2+2x+2\right)^2}dx=2\int\frac{1}{\left((x+1)^2+1\right)^2}dx=\begin{bmatrix}x+1=t\\dx=dt\end{bmatrix}=2\int\frac{1}{\left(t^2+1\right)^2}dt=$$

$$=2\frac{1}{2}\left(\frac{t}{1+t^2}+arctg\ t\right)+c_2=\underline{\underline{\frac{x+1}{x^2+2x+2}+arctg(x+1)+c_2}}$$

4. Najděte primitivní funkci k funkci $y=\dfrac{x^9}{x^8-2x^4+1}$ a určete její D_f.

$$y=\frac{x^9}{x^8-2x^4+1}=\frac{1}{2}\frac{2xx^8}{x^8-2x^4+1} \qquad D_f\in R-\{-1,1\}$$

$$\int\frac{x^9}{x^8-2x^4+1}dx=\begin{bmatrix}t=x^2\\ dt=2xdx\end{bmatrix}=\frac{1}{2}\int\frac{t^4}{t^4-2t^2+1}dt^*=$$

$$=\frac{1}{2}\left(\int dt+\frac{3}{4}\int\frac{1}{t-1}dt+\frac{1}{4}\int\frac{1}{(t-1)^2}dt-\frac{3}{4}\int\frac{1}{t+1}dt+\frac{1}{4}\int\frac{1}{(t+1)^2}dt\right)=$$

$$=\frac{1}{2}t+\frac{3}{8}\ln|t-1|-\frac{1}{8}\frac{1}{t-1}-\frac{3}{8}\ln|t+1|-\frac{1}{8}\frac{1}{t+1}+c=$$

$$=\underline{\underline{\frac{1}{2}x^2+\frac{3}{8}\ln\left|x^2-1\right|-\frac{1}{8}\frac{1}{x^2-1}-\frac{3}{8}\ln\left|x^2+1\right|-\frac{1}{8}\frac{1}{x^2+1}+c}}$$

$$*\int\frac{t^4}{t^4-2t^2+1}dt=\int\left(1+\frac{2t^2-1}{t^4-2t^2+1}\right)dt$$

$$\frac{2t^2-1}{t^4-2t^2+1}=\frac{2t^2-1}{(t^2-1)^2}=\frac{2t^2-1}{(t^2-1)(t^2+1)}=\frac{A}{t-1}+\frac{B}{(t-1)^2}+\frac{C}{t+1}+\frac{D}{(t+1)^2}$$

$$2t^2-1=A(t-1)(t+1)^2+B(t+1)^2+C(t+1)(t-1)^2+D(t-1)^2=$$

$$\vdots$$

$$=t^3(A+C)+t^2(A+B-C+D)+t(-A+2B-C-2D)+(-A+B+C+D)$$

$$\vdots$$

$$A=\frac{3}{4}$$

$$B=\frac{1}{4}=D$$

$$C=-\frac{3}{4}$$