Domácí úkoly z MAN2

Vypočtěte integrály:

1. $$\int \frac{\cos 2x}{\cos^2 x \sin^2 x} dx = \qquad \left(\sin x \cos x \neq 0,\ x \neq k\frac{\pi}{2},\ k\ celé \right)$$

$$= \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x} dx = \int \frac{1}{\sin^2 x} dx - \int \frac{1}{\cos^2 x} dx = \underline{\underline{-\cot x - \tan x + c}}$$

2. $$\int \frac{1}{\sin x \cos x} dx = \qquad \left(\sin x \cos x \neq 0,\ x \neq k\frac{\pi}{2},\ k\ celé \right)$$

$$= \int \frac{\frac{1}{\cos^2 x}}{\frac{\sin x \cos x}{\cos^2 x}} dx = \int \frac{\frac{1}{\cos^2 x}}{\tan x} dx = \underline{\underline{\ln|\tan x| + c}}$$

3. $$\int x^3 e^{x^4} dx = \qquad (x \in R)$$

$$\begin{bmatrix} x^4 = t \\ 4x^3 dx = dt \\ x^3 dx = \frac{dt}{4} \end{bmatrix}$$

$$= \frac{1}{4}\int e^t dt = \frac{e^t}{4} = \underline{\underline{\frac{e^{x^4}}{4}}} + c$$

4. $$\int \arctan x dx = \qquad (x \in R)$$

$$\begin{bmatrix} u' = 1 & v = \arctan x \\ u = x & v' = \frac{1}{1+x^2} \end{bmatrix}$$

$$= x \cdot \arctan x - \int x \frac{1}{1+x^2} dx =$$

$$\begin{bmatrix} 1 + x^2 = t \\ 2xdx = dt \\ xdx = \frac{dt}{2} \end{bmatrix}$$

$$= x \cdot \arctan x - \frac{1}{2}\int \frac{1}{t} dt = x \cdot \arctan x - \frac{1}{2}\ln|t| =$$

$$= \underline{\underline{x \cdot \arctan x - \frac{1}{2}\ln|1 + x^2| + c}}$$

5. $\int \frac{19x+15}{x^2-x-2}dx = \qquad (x \in R - \{-1, 2\})$

$$\left[\begin{array}{c} \frac{19x+15}{x^2-x-2} = \frac{A}{x+1} + \frac{B}{x-2} \\ 19x+15 = A(x-2) - B(x+1) \\ 19x+15 = x(A+B) - 2A + B \\ 19 = A + B \\ 15 = -2A + B \\ A = \frac{4}{3} \qquad B = \frac{53}{3} \end{array}\right]$$

$$= \frac{4}{3}\int \frac{1}{x+1}dx + \frac{53}{3}\int \frac{1}{x-2}dx = \underline{\underline{\frac{4}{3}\ln|x+1| + \frac{53}{3}\ln|x-2| + c}}$$

6. $\int \frac{2x^5-3x^2+1}{x^2-x-2}dx = \qquad (x \in R - \{-1, 2\})$

$$\left[\begin{array}{l} (2x^5 - 3x^2 + 1) : (x^2 - x - 2) = 2x^3 + 2x^2 + 6x + 7 + \frac{19x+15}{x^2-x-2} \\ -2x^5 \mp 2x^4 \mp 4x^3 \\ \qquad 2x^4 + 4x^3 - 3x^2 \\ \qquad -2x^4 \mp 2x^3 \mp 4x^2 \\ \qquad\qquad 6x^3 + x^2 \\ \qquad\qquad -6x^3 \mp 6x^2 \mp 12x \\ \qquad\qquad\qquad 7x^2 + 12x + 1 \\ \qquad\qquad\qquad -7x^2 \mp 7x \mp 14 \\ \qquad\qquad\qquad\qquad 19x + 15 \end{array}\right]$$

$$= 2\int x^3 dx + 2\int x^2 dx + 6\int x dx + 7\int dx + \int \frac{19x+15}{x^2-x-2}dx =$$

$$= \underline{\underline{\frac{x^4}{2} + \frac{2x^3}{3} + 3x^2 + 7x + \frac{4}{3}\ln|x+1| + \frac{53}{3}\ln|x-2| + c}}$$

7. $\int \frac{1}{x^2 - x + 1} dx =$ $(x \in R)$

$$= \frac{1}{\sqrt{1-\left(\frac{1}{2}\right)^2}} \arctan \frac{x - \frac{1}{2}}{\sqrt{1-\left(\frac{1}{2}\right)^2}} = \frac{2}{\sqrt{3}} \arctan \frac{2x-1}{\sqrt{3}} = \underline{\underline{\frac{2\sqrt{3}}{3} \arctan \frac{\sqrt{3}(2x-1)}{3} + c}}$$

8. $\int \frac{2x+3}{2x^3+2} dx = \frac{1}{2} \int \frac{2x+3}{x^3+1} dx =$ $(x \in R)$

$$\left[\begin{array}{l} \frac{2x+3}{x^3+1} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1} \\ 2x+3 = Ax^2 - Ax + A + Bx^2 + Bx + Cx + C \\ 2x+3 = x^2(A+B) + x(-A+B+C) + A + C \\ \quad 0 = A + B \\ \quad 2 = -A + B + C \\ \quad 3 = A + C \\ \quad A = \frac{1}{3} \qquad B = -\frac{1}{3} \qquad C = \frac{8}{3} \end{array}\right]$$

$$= \frac{1}{3} \int \frac{1}{x+1} dx + \frac{1}{3} \int \frac{8-x}{x^2-x+1} dx =$$

$$= \frac{1}{3} \int \frac{1}{x+1} dx - \frac{1}{6} \int \frac{2x-1}{x^2-x+1} dx + \frac{1}{6} \int \frac{15}{x^2-x+1} dx =$$

$$= \underline{\underline{\frac{1}{3} \ln|x+1| - \frac{1}{6} \ln|x^2-x+1| + \frac{5\sqrt{3}}{3} \arctan \frac{\sqrt{3}(2x-1)}{3} + c}}$$